INTEGER PROGRAMMING PROBLEMS (IPP)

METHOD 1: GOMORY'S ALL I.P.P (or) FRACTIONAL CUT Method

Step:1 Express the given L.P.P into its standard form and determine an optimum solution by using simplex method ignoring the integer value restriction.

Step2: Test the integrality of the optimum solution.

(a) If the optimum solution admits all-integer values, an optimum solution is attained.

b) otherwise go to next step.

Step 3: Choose the largest fractional value of the ~~basic~~ optimal solution. Let it be f_{k0} [k^{th} row]

Step4: Express each of the negative fractions if any, in the k^{th} row of the optimum simplex table as the sum of a negative integer and positive fraction

Step5: Generate the Gomorian Constraint (fractional cut) in the form

$$G_1 = -f_{k0} + f_{k1}x_1 + f_{k2}x_2 + \cdots + f_{kn}x_n.$$

where $0 \leq f_{kj} < 1$ and $0 < f_{k0} < 1$.

Step6: Add the Gomorian constraint generated Step5 at the bottom of the optimum table. Use dual simplex method to find an optimum solution.

Step7: Go to Step2 and repeat the procedure until an optimal all integer solution is obtained.

① Solve the following integer programming problem
Maximize $Z = 5x_1 + 4x_2$, Subject to the Constraints
$4x_1 + 5x_2 \le 10$, $3x_1 + 2x_2 \le 9$, $8x_1 + 3x_2 \le 12$
and $x_1, x_2 \ge 0$ and are integers.

Ex 2.1.1 using simplex method. optimum table.

	x_1	x_2	S_1	S_2	S_3	Solution	
Z	0	0	$\frac{17}{28}$	0	$\frac{9}{28}$	$\frac{139}{14}$	
x_2	0	1	$2/7$	0	$-1/7$	$8/7$	$1\frac{1}{7}$
S_2	0	0	$-\frac{1}{4}$	1	$-\frac{1}{4}$	$\frac{7}{2}$	$3\frac{1}{2}$
x_1	1	0	$-\frac{3}{28}$	0	$\frac{5}{28}$	$\frac{15}{14}$	$1\frac{1}{14}$

$x_1 = \frac{15}{14}$, $x_2 = \frac{8}{7}$ $\quad$ max $Z = \frac{139}{14}$

But x_1 and x_2 are not an integers.

$\therefore$ select ~~max~~ largest fractional part of Basic Var solutions $\therefore \max\left\{\frac{1}{7}, \frac{1}{2}, \frac{1}{14}\right\} = \frac{1}{2}$ corresponding to S_2 row $= f_{20}$

Introduce Gomorian Constraint

$G_1 = -f_{k0} + f_{k1}x_1 + f_{k2}x_2 + \cdots$

Here $k=2$ (since second row in the above table)

$\therefore G_1 = -f_{20} + f_{21}x_1 + f_{22}x_2 + f_{23}S_1 + f_{24}S_2 + f_{25}S_3$

Here $f_{20} = \frac{1}{2}$, $f_{21} = 0$ $f_{22} = 0$ $-\frac{1}{4} = -1 + \frac{3}{4}$ $\therefore f_{23} = \frac{3}{4}$

$f_{24} = 0$ $\quad f_{25} = \frac{3}{4}$ $\quad$ sub in the above eqn

$\therefore G_1 = -\frac{1}{2} + \frac{3}{4}S_1 + \frac{3}{4}S_3$

$-\frac{3}{4}S_1 - \frac{3}{4}S_3 + G_1 = -\frac{1}{2}$

Add the Gomorian Constraint at the bottom of the optimum table.

4^{th} Iteration ↓P.C

	x_1	x_2	S_1	S_2	S_3	G_1	Soln
Z	0	0	$\frac{17}{28}$	0	$\frac{9}{28}$	0	$\frac{139}{14}$
x_2	0	1	$\frac{2}{7}$	0	$\frac{-1}{7}$	0	$\frac{8}{7}$
S_2	0	0	$\frac{-1}{4}$	1	$\frac{-1}{4}$	0	$\frac{7}{2}$
x_1	1	0	$\frac{-3}{28}$	0	$\frac{5}{28}$	0	$\frac{15}{14}$
G_1	0	0	$\frac{-3}{4}$	0	$\frac{-3}{4}$	1	$-\frac{1}{2}$ ← PR

Pivot element

Apply Dual Simplex method

Select G_1 row corresponding row is pivot row

$\therefore$ G_1 leaves variable.

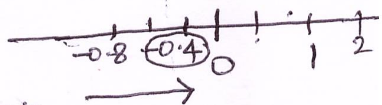
now to find pivot column.

Select, Max $\left\{ \frac{Z\text{-coefft}}{-\text{ve coefft of pivot row}} \right\}$

$$\therefore \text{Max} \left\{ \frac{\frac{17}{28}}{-\frac{3}{4}}, \frac{\frac{9}{28}}{\frac{-3}{4}} \right\} = \text{Max} \left\{ -\frac{17}{21}, -\frac{9}{21} \right\}$$

$$= \text{Max} \{ -0.8, -0.42 \}$$

$$= -0.42 \text{ Corr. to } S_3$$



$\therefore$ Corresponding column pivot column

$\therefore$ S_3 entering variable.

5^{th} Iteration

	x_1	x_2	S_1	S_2	S_3	G_1	Soln	
Z	0	0	$\frac{2}{7}$	0	0	$\frac{3}{7}$	$\frac{68}{7}$	
x_2	0	1	$\frac{3}{7}$	0	0	$-\frac{4}{21}$	$\frac{26}{21}$	$1\frac{5}{21}$
S_2	0	0	0	1	0	$-\frac{1}{3}$	$11/3$	$3\frac{2}{3}$
x_1	1	0	$-\frac{2}{7}$	0	0	$\frac{5}{21}$	$\frac{20}{21}$	$\frac{20}{21}$
New Pivot row → S_3	0	0	1	0	1	$-\frac{4}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

Again x_1 and x_2 are not an integers

Calculations z-coefficients	x_2 coeff	S_2 coeffts
$=0-\left(\frac{9}{28}\cdot 0\right)=0$	$=0+\frac{1}{7}\cdot 0=0$	$=0+\frac{1}{4}\cdot 0=0$
$=0-\left(\frac{9}{28}\cdot 0\right)=0$	$=1+\frac{1}{7}\cdot 0=1$	$=0+\frac{1}{4}\cdot 0=0$
$=\frac{17}{28}-\left(\frac{9}{28}\cdot 1\right)=\frac{8}{28}=\frac{2}{7}$	$=\frac{2}{7}+\frac{1}{7}\cdot 1=\frac{3}{7}$	$=-\frac{1}{4}+\frac{1}{4}\cdot 1=0$
$=0-\left(\frac{9}{28}\cdot 0\right)=0$	$=0+\frac{1}{7}\cdot 0=0$	$=1+\frac{1}{4}\cdot 0=1$
$=\frac{9}{28}-\left(\frac{9}{28}\cdot 1\right)=0$	$=-\frac{1}{7}+\frac{1}{7}\cdot 1=0$	$=-\frac{1}{4}+\frac{1}{4}\cdot 1=0$
$=0+\frac{9}{28}\left(\frac{4}{3}\right)=\frac{3}{7}$	$=0+\frac{1}{7}\left(-\frac{4}{3}\right)=-\frac{4}{21}$	$=0+\frac{1}{4}\left(-\frac{4}{3}\right)=-\frac{1}{3}$
$=\frac{139}{14}-\frac{9}{28}\left(\frac{2}{3}\right)=\frac{68}{7}$	$=\frac{8}{7}+\frac{1}{7}\cdot\frac{2}{3}=\frac{26}{21}$	$=\frac{7}{2}+\frac{1}{4}\left(\frac{2}{3}\right)=\frac{21+1}{6}=\frac{11}{3}$

$\therefore$ Introduce another Gomorian Constraint

$\therefore$ Select max fractinal Value of basic Variables

$$\therefore \max\left\{\frac{5}{21}, \frac{2}{3}, \frac{20}{21}, \frac{2}{3}\right\}=\frac{20}{21}$$

$$\max\{0.23, 0.6, 0.95, 0.6\}=0.95 \quad (\text{corr } x_1)$$

$\therefore$ corresponding to x_1
3rd row in the last table.

$$\therefore f_{30}=\frac{20}{21}$$

$$G_{12}=-f_{30}+f_{31}x_1+f_{32}x_2+f_{33}S_1+f_{34}S_2+f_{35}S_3+f_{36}G_1$$

Here $f_{31}=0$, $f_{32}=0$, $f_{33}=\frac{5}{7}$ $\quad \left(-1+\frac{5}{7}=-\frac{2}{7}\right)$

$f_{34}=0 \quad f_{35}=0 \quad f_{36}=\frac{5}{21}$

$$\therefore G_{12}=-\frac{20}{21}+\frac{5}{7}S_1+\frac{5}{21}G_1$$

$$-\frac{5}{7}S_1-\frac{5}{21}G_1+G_2=-\frac{20}{21}$$

Add this constraint in the last table.

↓ P.C

	x_1	x_2	s_1	s_2	s_3	G_1	G_2	Soln
Z	0	0	$\frac{2}{7}$	0	0	$\frac{3}{7}$	0	$\frac{68}{7}$
x_2	0	1	$\frac{3}{7}$	0	0	$-\frac{4}{21}$	0	$\frac{26}{21}$
s_2	0	0	0	1	0	$-\frac{1}{3}$	0	$\frac{11}{3}$
x_1	1	0	$-\frac{2}{7}$	0	0	$\frac{5}{21}$	0	$\frac{20}{21}$
s_3	0	0	1	0	1	$-\frac{4}{3}$	0	$\frac{2}{3}$
G_2	0	0	$-\frac{5}{7}$ PE	0	0	$-\frac{5}{21}$	1	$-\frac{20}{21}$ ← PR

Apply dual simplex method.
I^{st} select −ve soln row corresponding row pivot row.

$\therefore$ G_2 leaves Variable.

To find pivot Column, Find max $\left\{ \frac{\text{z-coefft}}{\text{-ve coefft PR}} \right\}$

$$\therefore \text{Max} \left\{ \frac{\frac{2}{7}}{-\frac{5}{7}}, \frac{\frac{3}{7}}{-\frac{5}{21}} \right\} = \text{Max} \left\{ -\frac{2}{5}, -\frac{9}{5} \right\}$$

$= -\frac{2}{5}$ corr to s_1 column

$\therefore$ s_1 entering Variable.

② Find the optimum integer solution to the following I.P.P, Maximize $Z = x_1 + 2x_2$

Subject to the constraints

$$2x_2 \le 7$$
$$x_1 + x_2 \le 7$$
$$2x_1 \le 11, \quad x_1, x_2 \ge 0 \text{ and are integers.}$$

Solution: Apply simplex method.

$$Z - x_1 - 2x_2 = 0$$
$$2x_2 + S_1 = 7$$
$$x_1 + x_2 + S_2 = 7$$
$$2x_1 + S_3 = 11$$

Let x_1, x_2 non basic variables and let $x_1 = x_2 = 0$

$\therefore S_1 = 7, \ S_2 = 7, \ S_3 = 11$

Ist - Iteration

least -ve PC

	x_1	x_2	S_1	S_2	S_3	Soln	
Z	−1	−2	0	0	0	0	PR
S_1	0	2 P.E	1	0	0	7	$\frac{7}{2} = 3.5$ least +ve ratio
S_2	1	1	0	1	0	7	$\frac{7}{1} = 7$
S_3	2	0	0	0	1	11	$\frac{11}{0} = \infty$

$\therefore x_2$ entering variable

S_1 leaves variable.

New pivot row $= \dfrac{\text{Old Pivot row}}{\text{Pivot element}}$ $\quad 0 \quad 1 \quad \frac{1}{2} \quad 0 \quad 0 \quad \frac{7}{2}$

Table 2 ↓ P.C

	x_1	x_2	S_1	S_2	S_3	soln	
Z	-1	0	1	0	0	7	
New Pivot row → x_2	0	1	$\frac{1}{2}$	0	0	$7/2$	
S_2	1 P.E	0	$-\frac{1}{2}$	1	0	$\frac{7}{2}$	$\frac{7}{2}$ least +ve ← PR
S_3	2	0	0	0	1	11	$\frac{11}{2}$

$\therefore$ x_1 entering Variable S_2 leaves Variable.

	x_1	x_2	S_1	S_2	S_3	G_1	soln	
Z	0	0	$\frac{1}{2}$	1	0	0	$\frac{21}{2}$	
x_2	0	1	$\frac{1}{2}$	0	0	0	$7/2$	$3\frac{1}{2}$
New Pivot row → x_1	1	0	$-\frac{1}{2}$	1	0	0	$7/2$	$3\frac{1}{2}$
S_3	0	0	1	-2	1	0	4	
G_1	0	0	$-\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	

All the Z-coeffts are ≥ 0 $\therefore$ above table is optimal table.

optimal solution $x_1 = 7/2$, $x_2 = 7/2$

$$\max Z = \frac{21}{2}$$

But x_1 and x_2 are not an integers.

$\therefore$ Apply Gomorian cutting plane method.

max fractional part of $\{\frac{1}{2}, \frac{1}{2}\} = \frac{1}{2} = f_{10}$

both are same, arbitrarly select x_2 row

$\therefore$ $G_1 = -f_{10} + f_{11}x_1 + f_{12}x_2 + f_{13}S_1 + f_{14}S_2 + f_{15}S_3$

$G_1 = -\frac{1}{2} + \frac{1}{2}S_1 \qquad -\frac{1}{2}S_1 + G_1 = -\frac{1}{2}$

$$-\frac{1}{2}S_1 + G_1 = -\frac{1}{2}$$

↓PC

	x_1	x_2	S_1	S_2	S_3	G_1	Soln
Z	0	0	$\frac{1}{2}$	1	0	0	$\frac{21}{2}$
x_2	0	1	$\frac{1}{2}$	0	0	0	$\frac{7}{2}$
x_1	1	0	$-\frac{1}{2}$	1	0	0	$\frac{7}{2}$
S_3	0	0	1	−2	1	0	4
G_1	0	0	$-\frac{1}{2}$ PE	0	0	1	$-\frac{1}{2}$ ← PR

∴ G_1 leaves Variable.

To Find Pivot column.

$$\max\left\{\frac{z\text{-coefft}}{-\text{ve coefft of PR}}\right\} = \max\left\{\frac{\frac{1}{2}}{-\frac{1}{2}}\right\} = -1$$

Corresponding to S_1 ∴ S_1 ~~to~~ entering Variable.

	x_1	x_2	S_1	S_2	S_3	G_1	Soln
Z	0	0	0	1	0	1	10
x_2	0	1	0	0	0	1	3
x_1	1	0	0	1	0	−1	4
S_3	0	0	0	−2	1	2	3
New Pivot row → S_1	0	0	1	0	0	−2	1

Z-Coeffts
$= 0-\frac{1}{2}\cdot 0 = 0$
$= 0-\frac{1}{2}\cdot 0 = 0$
$= \frac{1}{2}-\frac{1}{2}\cdot 1 = 0$
$= 1-\frac{1}{2}\cdot 0 = 1$
$= 0-\frac{1}{2}\cdot 0 = 0$
$= 0-\frac{1}{2}(-2) = 1$
$= \frac{21}{2}-\frac{1}{2}\cdot 1 = 10$

x_2 coeffts
$= 0-\frac{1}{2}\cdot 0 = 0$
$= 1-\frac{1}{2}\cdot 0 = 1$
$= \frac{1}{2}-\frac{1}{2}\cdot 1 = 0$
$= 0-\frac{1}{2}\cdot 0 = 0$
$= 0-\frac{1}{2}\cdot 0 = 0$
$= 0-\frac{1}{2}(-2) = 1$
$= \frac{7}{2}-\frac{1}{2}\cdot 1 = 3$

x_1-Coeffts
$= 1+\frac{1}{2}\cdot 0 = 1$
$= 0+\frac{1}{2}\cdot 0 = 0$
$= -\frac{1}{2}+\frac{1}{2}\cdot 1 = 0$
$= 1+\frac{1}{2}\cdot 0 = 1$
$= 0+\frac{1}{2}\cdot 0 = 0$
$= 0+\frac{1}{2}(-2) = -1$
$= \frac{7}{2}+\frac{1}{2}\cdot 1 = 4$

S_3-coeffts
$= 0-1\cdot 0 = 0$
$= 0-1\cdot 0 = 0$
$= 1-1\cdot 1 = 0$
$= -2-1\cdot 0 = -2$
$= 1-1\cdot 0 = 1$
$= 0-1(-2) = 2$
$= 4-1\cdot 1 = 3$

In the above table $x_1 = 4$, $x_2 = 3$ Max $Z = 10$
here x_1 and x_2 are an integers, $\therefore$ optimal solution.

Mixed Integer Programming problems
Gomorian cutting plane method.

Here Gomorian constraint is

$$G_i = -f_{ko} + \sum_{f_{kj} \in R_+} f_{kj} x_j + \left(\frac{f_{ko}}{f_{ko}-1}\right) \sum_{f_{kj} \in R_-} f_{kj} x_j .$$

suppose fractional Value $-\frac{5}{6}$

In all integer programming problem $-1 + \frac{1}{6} = -\frac{5}{6}$

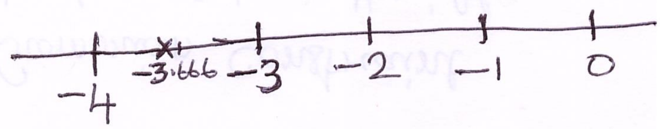
$\therefore f_{kj} = \frac{1}{6}$

But in mixed integer Prog Problem

$f_{kj} = -\frac{5}{6}$, $f_{kj} \in R_-$

Suppose fractional Value $-\frac{11}{3}$

$-\frac{11}{3} = -3.666$



In All integer prog pbm

$-\frac{11}{3} = -4 + \frac{1}{3}$ $\therefore f_{kj} = \frac{1}{3}$

In mixed integer prog pbm

$-\frac{11}{3} = -3 - \frac{2}{3}$ $\therefore f_{kj} = -\frac{2}{3}$ $f_{kj} \in R_-$

Note: Choose the largest fraction value among the basic variables which are **restricted to integers**.

① Solve the following mixed integer programming problem Maximize $Z = 4x_1 + 6x_2 + 2x_3$ subject to the constraints $4x_1 - 4x_2 \le 5$

$-x_1 + 6x_2 \le 5$

$-x_1 + x_2 + x_3 \le 5$

$x_1, x_2, x_3 \ge 0$, x_1 and x_3 are integers.

Solution: Apply simplex method.

$Z - 4x_1 - 6x_2 - 2x_3 = 0$

$4x_1 - 4x_2 + S_1 = 5$

$-x_1 + 6x_2 + S_2 = 5$

$-x_1 + x_2 + x_3 + S_3 = 5$

Let non basic variables are x_1, x_2, x_3 and

let $x_1 = x_2 = x_3 = 0$

$\therefore S_1 = 5$, $S_2 = 5$, $S_3 = 5$ basic feasible solution

I^{st} select pivot column

Table I

least -ve PC (x_2)

	x_1	x_2	x_3	S_1	S_2	S_3	soln	
Z	-4	-6	-2	0	0	0	0	
S_1	4	-4	0	1	0	0	5	
S_2	-1	6_{PE}	0	0	1	0	5	$\frac{5}{6}$ ← least +ve
S_3	-1	1	1	0	0	1	5	$\frac{5}{1}$

$\therefore x_2$ entering variable

S_2 leaves variable

$$\text{New pivot row} = \frac{\text{old pivot row}}{\text{Pivot element}} \quad \therefore \frac{-1}{6}, 1, 0, 0, \frac{1}{6}, 0, \frac{5}{6}$$

Table II — Pivot column — Select pivot column

	x_1	x_2	x_3	S_1	S_2	S_3	Soln
Z	-5	0	-2	0	1	0	5
S_1	$\frac{10}{3}$ PE	0	0	1	$\frac{2}{3}$	0	$\frac{25}{3}$
New Pivot row → x_2	$-\frac{1}{6}$	1	0	0	$\frac{1}{6}$	0	$\frac{5}{6}$
S_3	$-\frac{5}{6}$	0	1	0	$-\frac{1}{6}$	1	$\frac{25}{6}$

$\therefore$ x_1 entering Variable, S_1 leaves Variable

Table III — PC least -ve — Select pivot column

	x_1	x_2	x_3	S_1	S_2	S_3	Soln	
Z	0	0	-2	$\frac{3}{2}$	2	0	$\frac{35}{2}$	
NPR → x_1	1	0	0	$\frac{3}{10}$	$\frac{1}{5}$	0	$\frac{5}{2}$	$\frac{5/2}{0}=\infty$
x_2	0	1	0	$\frac{1}{20}$	$\frac{1}{5}$	0	$5/4$	$\frac{5/4}{0}=\infty$
S_3	0	0	1 PE	$\frac{1}{4}$	0	1	$\frac{25}{4}$	$\frac{25/4}{1}$ ←

$\therefore$ x_3 entering Var, S_3 leaves Variable

Table IV

	x_1	x_2	x_3	S_1	S_2	S_3	Soln	
Z	0	0	0	2	2	2	30	
x_1	1	0	0	$\frac{3}{10}$	$\frac{1}{5}$	0	$\frac{5}{2}$	$2\frac{1}{2}$
x_2	0	1	0	$\frac{1}{20}$	$\frac{1}{5}$	0	$\frac{5}{4}$	
NPR → x_3	0	0	1	$\frac{1}{4}$	0	1	$\frac{25}{4}$	$6\frac{1}{4}$

above table is optimal table using simplex method since Z-coeffts are ≥ 0.

$x_1 = \frac{5}{2}$ $\quad x_2 = \frac{5}{4}$ $\quad x_3 = \frac{25}{4}$ $\quad$ max $Z = 30$

But the restricted Variables x_1 and x_3 are not an integer $\therefore$ Apply Gomorian Cutting plane method for mixed integers

Select maximum fractional Value among x_1 and x_3 only $\text{Max}\left\{\overset{x_1}{\frac{1}{2}}, \overset{x_3}{\frac{1}{4}}\right\} = \frac{1}{2}$ corr to x_1, Ist row $\therefore$ Let $f_{10} = \frac{1}{2}$

Select x_1 row, $\because$ all the coeffts are 0 and +ve fractions

$$G_1 = -f_{10} + f_{11}x_1 + f_{12}x_2 + f_{13}x_3 + f_{14}S_1 + f_{15}S_2 + f_{16}S_6$$

Here $f_{11}=0$, $f_{12}=0$, $f_{13}=0$, $f_{14}=\frac{3}{10}$, $f_{15}=\frac{1}{5}$, $f_{16}=0$

$$\therefore G_1 = -\frac{1}{2} + \frac{3}{10}S_1 + \frac{1}{5}S_2$$

$$\therefore -\frac{3}{10}S_1 - \frac{1}{5}S_2 + G_1 = -\frac{1}{2}$$

add this constrain in the last optimum table.

Table V

	x_1	x_2	x_3	S_1 (PC)	S_2	S_3	G_1	soln
Z	0	0	0	2	2	2	0	30
x_1	1	0	0	$\frac{3}{10}$	$\frac{1}{5}$	0	0	$\frac{5}{2}$
x_2	0	1	0	$\frac{1}{20}$	$\frac{1}{5}$	0	0	$\frac{5}{4}$
x_3	0	0	1	$\frac{1}{4}$	0	1	0	$\frac{25}{4}$
G_1	0	0	0	$\frac{-3}{10}$ (PE)	$\frac{-1}{5}$	0	1	$-\frac{1}{2}$ ← PR

Apply Dual simplex method. Ist select Pivot row (select −ve soln corr row PR)

$\therefore G_1$ leaves Variable

To Find Pivot Column

Select $\max\left\{\frac{z\text{-coefft}}{-\text{ve coefft of PR}}\right\} = \max\left\{\frac{2}{-\frac{3}{10}}, \frac{2}{-\frac{1}{5}}\right\}$

$= \max\left\{\overset{S_1}{-\frac{20}{3}}, \overset{S_2}{-10}\right\} = \max\left\{\underset{\substack{\max\\S_1}}{-6.66}, \underset{S_2}{-10}\right\}$

Corr to S_1 $\therefore$ Corr column pivot column

$\therefore S_1$ entering Variable

Table $\overline{VI}$

	x_1	x_2	x_3	S_1	S_2	S_3	G_1	G_2	soln	
Z	0	0	0	0	$\frac{2}{3}$	2	$\frac{20}{3}$	0	$\frac{80}{3}$	
x_1	1	0	0	0	0	0	1	0	2	
x_2	0	1	0	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{7}{6}$	
x_3	0	0	1	0	$-\frac{1}{6}$	1	$\frac{5}{6}$	0	$\frac{35}{6}$	$5\frac{5}{6}$
S_1	0	0	0	1	$\frac{2}{3}$	0	$-\frac{10}{3}$	0	$\frac{5}{3}$	
G_2	0	0	0	0	$-\frac{5}{6}$	0	$-\frac{5}{6}$	1	$-\frac{5}{6}$	← PR

PE

Here $x_1 = 2$, $x_2 = \frac{7}{6}$ $x_3 = \frac{35}{6}$ max $Z = \frac{80}{3}$

Since x_3 is still not an integer,

$\therefore$ Select corr row to form Gomorian constraint

$$G_2 = -f_{30} + f_{37} G_1 + \left(\frac{f_{30}}{f_{30}-1}\right)(f_{35}) S_2 \quad \boxed{3^{rd} \text{ row} \quad \therefore f_{30} = \frac{5}{6}}$$

$$\left[G_i = -f_{k0} + \sum_{f_{kj} \in R_+} f_{kj} x_j + \left(\frac{f_{k0}}{f_{k0}-1}\right) \sum_{f_{kj} \in R_-} f_{kj} x_j \right]$$

$$\therefore G_2 = -\frac{5}{6} + \frac{5}{6} G_1 + \left(\frac{\frac{5}{6}}{\frac{5}{6}-1}\right)\left(-\frac{1}{6}\right) S_2$$

$$\frac{\left(\frac{5}{6}\right)}{\left(-\frac{1}{6}\right)}\left(-\frac{1}{6}\right)$$

$$G_2 = -\frac{5}{6} + \frac{5}{6} G_1 + \frac{5}{6} S_2$$

$$-\frac{5}{6} S_2 - \frac{5}{6} G_1 + G_2 = -\frac{5}{6}$$

add this constraint in the above table.

now apply Dual simplex method

Select pivot row first (ie) select –ve soln row)

~~next to~~ $\therefore G_2$ leaves variable

To find pivot column, max $\left\{\frac{z\text{-coefft}}{-\text{ve coefft of PR}}\right\}$

$$\max\left\{\frac{\frac{2}{3}}{\frac{-5}{6}}, \frac{\frac{20}{3}}{\frac{-5}{6}}\right\}$$

$$\max\left\{\underset{S_2}{-\frac{4}{5}}, \underset{G_1}{-8}\right\} = -\frac{4}{5} \text{ corr to } S_2$$

$\therefore$ S_2 entering Variable

	x_1	x_2	x_3	S_1	S_2	S_3	G_1	G_2	Soln
Z	0	0	0	0	0	2	$\frac{20}{3}$	$\frac{4}{5}$	26
x_1	1	0	0	0	0	0	1	0	2
x_2	0	1	0	0	0	0	$\frac{1}{6}$	$\frac{1}{5}$	1
x_3	0	0	1	0	0	1	$\frac{5}{6}$	$-\frac{1}{5}$	6
S_1	0	0	0	1	0	0	$\frac{-10}{3}$	$\frac{4}{5}$	1
S_2	0	0	0	0	1	0	1	$\frac{-6}{5}$	1

Here $x_1 = 2$, $x_2 = 1$, $x_3 = 6$

Max $Z = 26$ We got x_1 and x_3 are integers

$\therefore$ above solution optimal solution.

Assignment problems

① Maximize $Z = 7x_1 + 9x_2$, subject to the constraints $-x_1 + 3x_2 \le 6$; $7x_1 + x_2 \le 35$, $x_1, x_2 \ge 0$ and x_1 is an integer.

② Maximize $Z = -3x_1 + x_2 + 3x_3$ subj to the constraints $-x_1 + 2x_2 + x_3 \le 4$; $4x_2 - 3x_3 \le 2$; $x_1 - 3x_2 + 2x_3 \le 3$ x_1 and x_3 are integers and $x_j \ge 0$, $j = 1, 2, 3$